A Simple Method for Calculating the Reflection Coefficient of Open-Ended Waveguides

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***Abstract*— This paper presents a simple but effective method for the analysis of open-ended waveguides. The method begins with the introduction of a large waveguide to approximate the half-space. In order to avoid the convergence problem lossy dielectric is assumed to homogeneously fill the large waveguide. After obtaining a number of convergent data for different values of the loss tangent an extrapolation technique is employed to calculate the solution to the original problem—a waveguide terminated by an infinite conducting flange and radiating into a lossless or low-loss half-space. Numerical results are given to show the validity of the proposed method. The behavior of the effect of the loss tangent on the size of the large waveguide and on the final results are also examined.**

***Index Terms*—Modal analysis, open-ended waveguide, waveguide junction.**

I. Introduction

Open-ended waveguides have found wide applications in aeronautics, large phased array systems, thermography, diathermy and hyperthermia, and the measurement of material properties, etc. Theoretical and experimental studies of open-ended waveguides have occupied the attention of numerous researchers for several decades (see [1]). The variational principle [2], the correlation matrix method [3], and the transverse operator method [4] were used to compute the aperture admittance of an open-ended rectangular waveguide with infinite flange. The radiation from the TEM mode in a coaxial line terminated with a flat infinite metal plate was considered in [5] and later on was further studied and applied to nondestructive measurement of materials in [6]. The analysis and application of open-ended circular waveguides were then studied in [7].

In order to obtain accurate results for the aperture admittance of the flanged open-ended waveguide the rigorous full-wave analysis [3], [4], [6], [7] should be invoked. However, these methods are complicated and involve numerical integration, and therefore, are computationally expensive. It is desirable to develop a method which is simple and yet can give accurate solutions to this classical problem.

The idea of the method presented in this paper is as follows. Firstly, a large waveguide is introduced to approximate the half-space. As pointed out in [8] the size of this waveguide should be very large when the medium in the half-space is low-loss or lossless (which results in expensive computational effort) since the authors must take a very large number of modes in the large waveguide into account to ensure convergent results. In order to overcome this drawback of the technique and to reduce the size of the large waveguide to save computer time, it is assumed to be filled with a homogeneous moderately lossy medium. The numerical results for lossy dielectric are then employed to calculate the solution to the actual lossless or low-loss half-space problem by an extrapolation technique. A similar idea, which was termed "complexification and extrapolation," was presented in [9] to accelerate the iterative solution of electromagnetic scattering problems.

Numerical tests demonstrate that the proposed method is simple but effective for open-ended waveguide problems. Numerical results for the reflection coefficient of an open-ended coaxial line and the aperture admittance of an open-ended rectangular waveguide are presented and compared with exact solutions in [3] and [6]. Very good agreement is obtained. It will be shown that the size of the large waveguide depends on the loss tangent of the medium assumed, while the resultant solutions are not sensitive to the assumed loss tangent of the medium in the large waveguide. This property makes the large waveguide be of moderate size to reduce computation time.

II. Description of the Proposed Method

Fig. 1(a) shows the side view of an open-ended waveguide with an infinite conducting flange, where the cross section of the waveguide can be arbitrary. At first, the authors introduce a large waveguide to approximate the half-space. Then the problem considered reduces to that of a waveguide junction as illustrated in Fig. 1(b). The cross section of the postulated large waveguide should be the same as, or similar to, that of the input waveguide (guide 1, Fig. 1) to simplify the analysis of the related waveguide junction. Meanwhile, symmetry may also be taken into account to reduce the computational complexity.

The study in [8] stated that the size of the large waveguide should be very large for a low-loss medium retained in the large waveguide. It was then concluded that the correlation matrix method [3] must be used for lossless and low-loss cases to increase accuracy and save computer time. Unfortunately, for most practical applications, the medium in the half-space is lossless or low loss. Therefore, it appears that the method of introducing a large waveguide to represent the half-space is not useful for most practical cases. However, the purpose of this paper is to demonstrate that this method can be useful for most practical applications by using the complexification and extrapolation technique [9]. It assumes that the large waveguide is homogeneously filled with a lossy dielectric (producing physical loss to eliminate the effect of the perfectly conducting wall of the large waveguide); the authors then *extrapolate* back to the lossless or low-loss case. The introduction of the lossy medium into the large waveguide makes it possible to use a moderately large guide to save computer time.

After obtaining the reflection coefficient or aperture admittance data for the waveguide junction for several different values of dielectric loss tangent, (i.e., three different values of loss tangent) an extrapolation technique (parabolic extrapolation) is employed to calculate the solution to the lossless or low-loss half-space problem in the following way. Suppose the imaginary parts of three complexified ϵ_{r2} values are ϵ''_{r21}, ϵ''_{r22}, and ϵ''_{r23} and the corresponding reflection coefficients are Γ_1, Γ_2, and Γ_3, respectively. The authors compute the quadratic Lagrange interpolation polynomial through the points $(\epsilon''_{r21}, \Gamma_1)$, $(\epsilon''_{r22}, \Gamma_2)$, and $(\epsilon''_{r23}, \Gamma_3)$ as follows:

$$\Gamma(\epsilon''_{r2}) = \frac{p_1\Gamma_1 + p_2\Gamma_2 + p_3\Gamma_3}{(\epsilon''_{r21} - \epsilon''_{r22})(\epsilon''_{r22} - \epsilon''_{r23})(\epsilon''_{r21} - \epsilon''_{r23})} \tag{1}$$

where

$$p_1 = (\epsilon''_{r22} - \epsilon''_{r23})(\epsilon''_{r2} - \epsilon''_{r22})(\epsilon''_{r2} - \epsilon''_{r23}) \tag{2}$$

$$p_2 = (\epsilon''_{r23} - \epsilon''_{r21})(\epsilon''_{r2} - \epsilon''_{r21})(\epsilon''_{r2} - \epsilon''_{r23}) \tag{3}$$

$$p_3 = (\epsilon''_{r21} - \epsilon''_{r22})(\epsilon''_{r2} - \epsilon''_{r21})(\epsilon''_{r2} - \epsilon''_{r22}) \tag{4}$$

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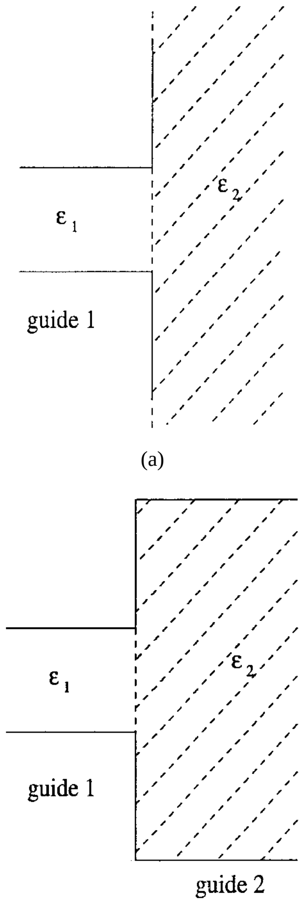
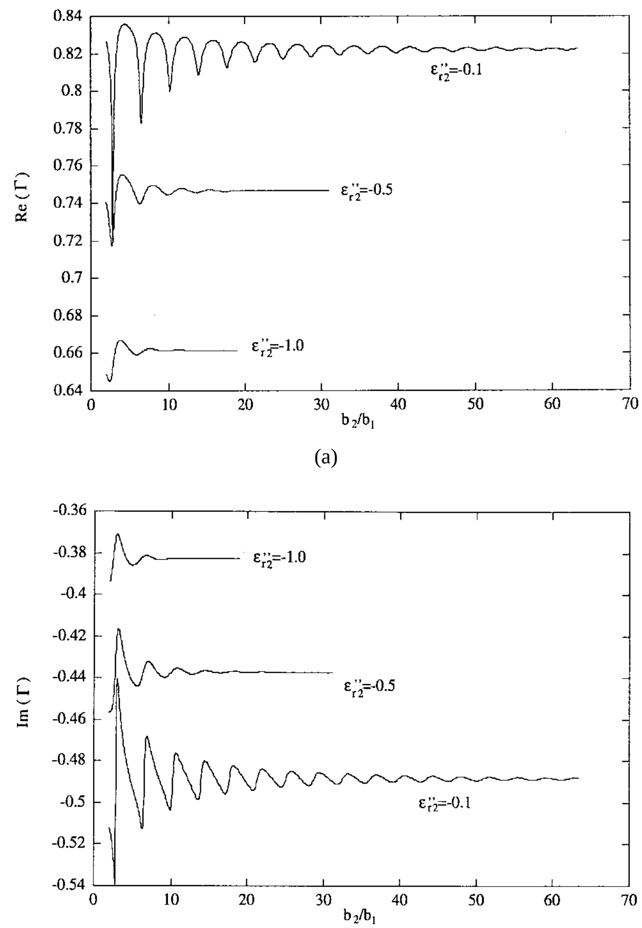


(a)

(b)

Fig. 1. (a) Side-view of an open-ended waveguide and its (b) simplified waveguide junction model.



(a)

(b)

Fig. 2. Variation of the reflection coefficient of an open-ended coaxial line with respect to the radius of the large circular waveguide for different values of loss tangent ($a_1 = 1.4364$ mm, $b_1 = 4.725$ mm, $\epsilon_{r1} = \epsilon'_{r2} = 2.05$, $f = 6$ GHz). (a) Real part of Γ. (b) Imaginary part of Γ.

and then set ϵ''_{r2} equal to zero, or to the actual small loss tangent, to yield the desired reflection coefficient of a waveguide terminated by an infinite conducting flange.

As pointed out in [9], parabolic extrapolation is second-order accurate provided the function $\Gamma(\epsilon''_{r2})$ is sufficiently smooth. This means that for uniform spaces $\Delta\epsilon''_{r2} = \epsilon''_{r23} - \epsilon''_{r22} = \epsilon''_{r22} - \epsilon''_{r21}$ the error in Γ due to the extrapolation is $0[(\Delta\epsilon''_{r2})^2]$. In practice, the authors have found that the accuracy of the extrapolation procedure is better than the estimate would suggest.

III. Numerical Examples

This section presents some numerical results for the reflection coefficient of an open-ended coaxial line and the aperture admittance of an open-ended rectangular waveguide by the method described in Section II. Firstly, the authors give some results for the effect of the loss tangent on the size of the large waveguide. Fig. 2 shows the variation of the reflection coefficient of a coaxial-to-circular waveguide junction with respect to the size of the large circular waveguide for different values of loss tangent. It is seen that the lower the loss of the medium retained in the large waveguide, the larger is its size to obtain convergent results. For the very lossy medium, i.e., $\epsilon_{r2} = 2.05 - j$, the result obtained with $b_2/b_1 = 7$ is quite satisfactory. The number of modes assumed in the large waveguide depends on the size of the waveguide according to the following equation:

$$N_2 = N_1 \frac{b_2}{b_1 - a_1} \tag{5}$$

where N_1 and N_2 are the numbers of modes considered in the input and large waveguides, a_1 and b_1 are the inner and outer radii of the coaxial line, and b_2 is the radius of the large circular waveguide. Results presented in Fig. 2 are obtained by fixing $N_1 = 10$ (the dominant TEM mode plus nine TM_{0n} modes).

Fig. 3 shows the comparison of results obtained by the proposed method and data given in [6] for the reflection coefficients of an open-ended coaxial line. It is seen that the agreement is excellent. The results shown in Fig. 3 are obtained by extrapolating three values of the reflection coefficient for $\epsilon_{r2} = 2.05 - j0.2$, $2.05 - j0.4$, and $2.05 - j0.6$ back to the real axis of the relative permittivity ϵ_{r2}.

The effect of different reference data used for extrapolation on the resultant reflection coefficient is examined in Table I, where the results obtained by extrapolating three groups of data are compared. It can be seen that the maximum error between the results for Case 1 and Case 2 is less than 0.1%, and the maximum difference of the absolute values of the reflection coefficients between Case 1 and Case 3 is less than 1%. The loss tangents assumed in Case 3 are quite big, which makes the size of the large circular waveguide only about ten times that of the coaxial line while the accuracy of the resultant reflection coefficient is still quite satisfactory. This shows that the method described in this paper is computationally efficient, very easy to implement, and also very accurate for the open-ended waveguide problems.

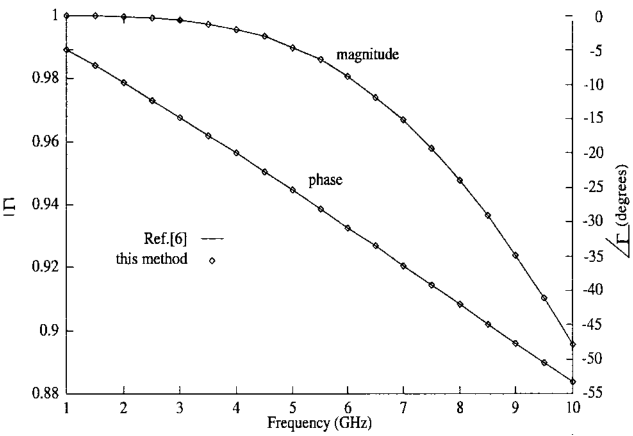


Fig. 3. Magnitude and phase of the reflection coefficient of a coaxial line terminated by an infinite flange ($a_1 = 1.4364$ mm, $b_1 = 4.725$ mm, $\epsilon_{r1} = \epsilon_{r2} = 2.05$).

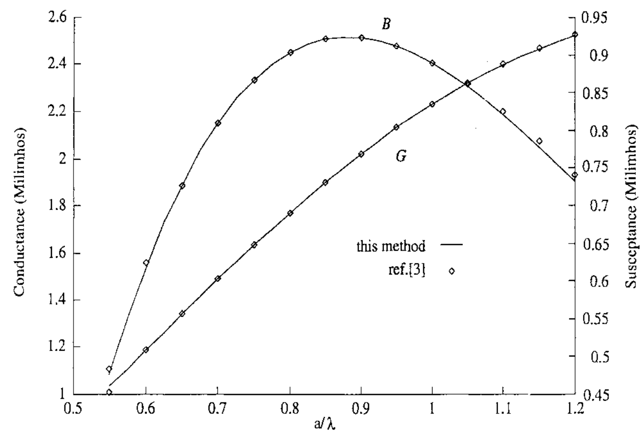


Fig. 4. Aperture admittance of an open-ended rectangular waveguide with infinite conducting flange ($a = 2.25b$, $\epsilon_{r1} = \epsilon_{r2} = 1$).

TABLE I

MAGNITUDE AND PHASE OF THE REFLECTION COEFFICIENT OF AN OPEN-ENDED COAXIAL LINE WITH INFINITE FLANGE ($a_1 = 1.4364$ mm, $b_1 = 4.725$ mm, $\epsilon_{r1} = \epsilon_{r2} = 2.05$). CASE 1: USING $\epsilon''_{r2} = -0.1, -0.2, -0.3$ THREE POINTS FOR EXTRAPOLATION; CASE 2: USING $\epsilon''_{r2} = -0.2, -0.4, -0.6$ THREE POINTS; CASE 3: USING $\epsilon''_{r2} = -0.5, -1.0, -1.5$ THREE POINTS

Frequency	Magnitude			Phase (degrees)		
(GHz)	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
1	0.99997	0.99998	1.00003	-4.8692	-4.8689	-4.8663
1.5	0.99992	0.99992	0.99995	-7.3204	-7.3210	-7.3183
2.0	0.99976	0.99972	0.99972	-9.8005	-9.7978	-9.7921
2.5	0.99920	0.99929	0.99924	-12.295	-12.298	-12.294
3.0	0.99874	0.99860	0.99840	-14.812	-14.834	-14.827
3.5	0.99732	0.99740	0.99705	-17.414	-17.405	-17.397
4.0	0.99570	0.99568	0.99508	-20.000	-20.012	-20.004
4.5	0.99347	0.99324	0.99233	-22.668	-22.660	-22.649
5.0	0.98984	0.98997	0.98868	-25.344	-25.344	-25.331
5.5	0.98604	0.98576	0.98402	-28.057	-28.064	-28.048
6.0	0.98062	0.98049	0.97824	-30.829	-30.817	-30.796
6.5	0.97420	0.97408	0.97128	-33.590	-33.597	-33.570
7.0	0.96685	0.96646	0.96308	-36.405	-36.400	-36.365
7.5	0.95779	0.95760	0.95362	-39.226	-39.219	-39.175
8.0	0.94786	0.94747	0.94292	-42.048	-42.048	-41.994
8.5	0.93651	0.93609	0.93098	-44.892	-44.881	-44.815
9.0	0.92387	0.92350	0.91787	-47.716	-47.710	-47.633
9.5	0.91025	0.90973	0.90365	-50.538	-50.530	-50.440
10.0	0.89534	0.89487	0.88841	-53.346	-53.333	-53.233

Finally, the authors present some calculated results for the aperture admittance of an open-ended rectangular waveguide with an infinite flange. For this problem the introduced large waveguide is assumed to be of rectangular shape and collinear with the input waveguide. The aperture admittance is defined as

$$Y = G + jB = Y_{0,\,10}\frac{1 - \Gamma_{10}}{1 + \Gamma_{10}} \tag{6}$$

where $Y_{0,\,10}$ and Γ_{10} are the characteristic admittance and the reflection coefficient of the dominant TE_{10} mode. In Fig. 4 the authors' results are compared with available data obtained by the correlation matrix method [3]. The authors' results shown in Fig. 4 are obtained by extrapolating three values of the aperture admittance for $\epsilon_{r2} = 1 - j0.1$, $1 - j0.2$, and $1 - j0.3$ to that for real ϵ_{r2}. It is noted that the agreement between them is very good, which also verifies the validity of the method presented in this paper.

IV. CONCLUSION

This paper has provided a simple but very effective method for determining the reflection coefficients and aperture admittances of open-ended waveguides. The method is based on the introduction of a large waveguide to approximate the half-space and on the complexification and extrapolation technique. It has been shown that the method proposed here is very easy to implement and can give quite accurate results. The value of the assumed loss tangent has a significant effect on the size of the second waveguide but does not influence the final result. The method introduced in this paper applies to open-ended waveguides of arbitrary cross section and can also be extended to analyze open-ended waveguides without, or with, finite flange, horn antennas, and open-ended waveguides radiating into stratified media, etc.

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